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PORTFOLIO OPTIMIZATION USING INTERVAL ANALYSIS

Abstract. *In this paper, a new model for solving portfolio optimization problems is proposed. Interval analysis and interval linear programming concepts are introduced and integrated in order to build an interval linear programming model. We develop an algorithm for solving portfolio optimization problems with the coefficients of the constraints and the coefficients of the objective function modeled by interval numbers. The theoretical results obtained are used to solve a case study.*

Keywords: *portfolio optimization, interval analysis, interval linear programming.*

JEL CLASSIFICATION: C02, C61, G11.

1. INTRODUCTION

The topic of portfolio management has been an area of special interest for researchers and practitioners for more than fifty years. Portfolio optimization models are based on the seminal work of Markowitz (1952), which uses the mean value of a random variable for assessing the return and the variance for estimating the risk. The original model for portfolio selection problems developed by Markowitz (1952) aims, on the one hand, to minimize the risk measure and on the other hand to ensure that the rate of return will be at least equal to a specified amount, according to the objectives of the decision maker. A new model for the portfolio selection problem was proposed by

Sharpe (1963). He introduced systematic risk, enabling the evaluation of the sensitivity of a stock's return to the market return. Other models were developed in order to reduce the issues arising from the portfolio selection problem, see, for example, Chiodi et al., 2003, Kellerer et al. (2000), Mansini et al. (2003), Michalowski and Ogryczak (2001), Papahristodoulou and Dotzauer (2004), and Rockafellar and Uryasev (2000).
Recently a lot of papers have developed different techniques for the optimization of the decision making process in finance and insurance, see, for example, Toma and Dedu, 2014, Ştefãnoiu et al., 2014, Preda et al., 2014, Toma, 2014, Paraschiv and Tudor, 2013, Dedu, 2012, Toma, 2012, Dedu and Ciumara, 2010 and Ştefãnescu et al., 2010. In the recent literature different approaches to solving the portfolio selection problem can be found, see, for example, Toma and Leoni-Aubin, 2013, Toma, 2012, Tudor, 2012, Şerban et al., 2011, Dedu and Fulga, 2011 and Ştefãnescu et al., 2008. In the majority of the current models for portfolio selection, return and risk are considered to be the two factors which are the most significant when it comes to determining the decisions of investors. There is, however, evidence that return and risk may not capture all the relevant information for portfolio selection. In the real world, when it comes to the portfolio selection problem, the exact and complete information related to various input parameters which decision makers need is not always available. That is to say, uncertainty is an intrinsic characteristic of real-world portfolio selection problems arising out of the distinct nature of the multiple sources which influence the economic phenomena. This uncertainty may be interpreted as randomness or fuzziness. In order to deal with randomness in multicriteria decision-making problems, techniques have been developed based on stochastic programming techniques, see, for example, Birge and Louveaux (1993). When looking at the second issue, in order to deal with fuzziness in multicriteria decision-making problems, Zadeh (1978) and Sakawa (1993) used fuzzy programming methods. In stochastic programming, the coefficients of the optimization problem are assumed to be imprecise in the stochastic sense and described by random variables with known probability distributions. As well as that, in fuzzy programming, the coefficients of the problem are described by using fuzzy numbers with known membership functions. The problem of specifying the distributions of random variables and membership functions of fuzzy numbers is fraught with difficulties. This is validated by the fact that sometimes the assigned parameters do not match real situations perfectly. Where such kinds of issues arise, interval programming may provide an alternative approach when attempting to deal with uncertainty in decision making problems. Interval programming offers some interesting characteristics since it does not require the specification or assumption of probabilistic distribution, as is the case in stochastic programming, or membership functions, as is the case in fuzzy programming, see, for example, Wu (2009). Interval programming just makes the assumption that information concerning the range of variation of some of the parameters is available,

which allows the development of a model with interval coefficients. Interval programming does not impose stringent applicability conditions; hence, it provides an interesting approach for modeling uncertainty in the objective functions or for modeling uncertainty in the constraints of a multicriteria decision-making problem, see, for example, Oliveira and Antunes (2009). A lot of authors have used and continue to use interval programming in order to address real-world problems.

In this paper we firstly present the fundamental concepts of interval analysis. Then method for solving a linear programming model with interval coefficients, known as interval linear programming, is proposed. Further we consider an interval portfolio selection problem with uncertain returns based on interval analysis and we propose an approach to reduce the interval programming problem with uncertain objective and constraints into two standard linear programming problems. Finally, using computational results, we prove that our method is capable of helping investors to find efficient portfolios according to their preferences.

2. INTERVAL ANALYSIS

The foundations of interval analysis were established and developed by Moore (1966, 1979). The capability of interval analysis to solve a wide variety of real life problems in an efficient manner enabled the extension of its concepts to the probabilistic framework. In this way, the classical concept of random variable was extended to cover the interval random variable concept, which allows the modeling not only of the randomness character, using the concepts of probability theory, but also the modeling of imprecision and non-specificity, using the concepts of interval analysis. The interval analysis based approach provides for the development of mathematical methods and computational tools that enable modeling data and solving optimization problems under uncertainty.

The results presented in this section are discussed in more detail in Alefeld and Herzberger (1983).

2.1. INTERVAL NUMBERS AND INTERVAL RANDOM VARIABLES

First we introduce the basic concept of interval number. Let $x^L, x^U \in \mathbf{R}$ be real numbers, with $x^L \le x^U$.

Definition 2.1. An *interval number* is a set defined as follows:

$$[x] = \left\{ x \in \mathbf{R} \,|\, x^L \le x \le x^U ;\; x^L, x^U \in \mathbf{R} \right\}.$$

We denote by $[x]$ the interval number $[x^L, x^U]$, with $x^L, x^U \in \mathbf{R}$.

Remark 2.1. If $x^L = x^U$, the interval number $[x^L, x^U]$ is said to be a *degenerate interval number*. Otherwise, this is said to be a *proper interval number*.

Remark 2.2. A real number $x \in \mathbf{R}$ can be regarded as the interval number $[x, x]$.

Definition 2.2. An interval number $x = [x^L, x^U]$ is said to be:

- *negative*, if $x^U < 0$; *positive*, if $x^L > 0$;
- *nonnegative*, if $x^L \geq 0$; *nonpositive*, if $x^U \leq 0$.

2.2. INTERVAL ARITHMETICS

Many relations and operations defined on sets or pairs of real numbers can be extended to operations on intervals. Let $x = [x^L, x^U]$ and $y = [y^L, y^U]$ be interval numbers.

Definition 2.3. The *equality* between interval numbers is defined as follows:

$$[x] = [y] \text{ if } x^L = y^L \text{ and } x^U = y^U.$$

Definition 2.4. The *median* of the interval number $x = [x^L, x^U]$ is defined by:

$$m(x) = \frac{x^L + x^U}{2}.$$

Definition 2.5. The *product between the real number a and the interval number* $[x]$ is defined by:

$$a \cdot [x] = \{a \cdot x | x \in [x]\} = \begin{cases} [a \cdot x^L, a \cdot x^U], & \text{if } a > 0 \\ [a \cdot x^U, a \cdot x^L], & \text{if } a < 0 \\ [0], & \text{if } a = 0 \end{cases}.$$

Let $x = [x^L, x^U]$ and $y = [y^L, y^U]$ be interval numbers.

Definition 2.6. The *summation* between two interval numbers is defined by:

$$[x] + [y] = [x^L + y^L, x^U + y^U].$$

Definition 2.7. The *subtraction* between two interval numbers is defined as follows:

$$[x] - [y] = [x^L - y^U, x^U - y^L].$$

Definition 2.8. The *product* between two interval numbers is defined by:

$$[x]\times[y]=\left[\min\left(x^L y^L, x^L y^U, x^U y^L, x^U y^U\right), \max\left(x^L y^L, x^L y^U, x^U y^L, x^U y^U\right)\right].$$

Definition 2.9. The inverse of the interval number $y=[y^L, y^U]$ is defined by:

$$\frac{1}{[y]}=\left[\frac{1}{y^U}, \frac{1}{y^L}\right], \text{ if } 0\notin[y].$$

Definition 2.10. The division between the interval numbers $x=[x^L, x^U]$ and $y=[y^L, y^U]$ is defined by:

$$\frac{[x]}{[y]}=[x]\times\frac{1}{[y]}=\left[\min\left(\frac{x^L}{y^U}, \frac{x^U}{y^L}\right), \max\left(\frac{x^L}{y^U}, \frac{x^U}{y^L}\right)\right], \text{ if } 0\notin[y].$$

It can be written, if $0\in[y]$ as follows:

$$\frac{[x]}{[y]}=\begin{cases}\left[\frac{x^U}{y^L}, \infty\right], & \text{if } x^U\le 0 \text{ and } y^U=0\\ \left[-\infty, \frac{x^U}{y^U}\right]\cup\left[\frac{x^U}{y^L}, \infty\right], & \text{if } x^U\le 0 \text{ and } y^L<0<y^U\\ \left[-\infty, \frac{x^U}{y^U}\right], & \text{if } x^U\le 0 \text{ and } y^L=0\\ [-\infty, \infty], & \text{if } x^L<0<x^U\\ \left[-\infty, \frac{x^L}{y^L}\right], & \text{if } x^L\ge 0 \text{ and } y^U=0\\ \left[-\infty, \frac{x^L}{y^L}\right]\cup\left[\frac{x^L}{y^U}, \infty\right], & \text{if } x^L\ge 0 \text{ and } y^L<0<y^U\\ \left[\frac{x^L}{y^U}, \infty\right], & \text{if } x^L\ge 0 \text{ and } y^L=0\end{cases}$$

$$[x]^n=\begin{cases}[1,1], & \text{if } n=0\\ \left[(x^L)^n, (x^U)^n\right], & \text{if } x^L\ge 0 \text{ or } n \text{ odd}\\ \left[(x^U)^n, (x^L)^n\right], & \text{if } x^U\le 0 \text{ and } n \text{ even}\\ \left[0, \max\left(\left[(x^L)^n, (x^U)^n\right]\right)\right], & \text{if } x^L\le 0\le x^U \text{ and } n>0 \text{ even}\end{cases}$$

2.3. INEQUALITIES BETWEEN INTERVAL NUMBERS

Now we will extend the classical inequality relations between real numbers to inequality relations between interval numbers.

Let $x = [x^L, x^U]$ and $y = [y^L, y^U]$ be interval numbers, with $x^L, x^U, y^L, y^U \in \mathbf{R}$.

Definition 2.13. We say that $[x] \le [y]$ if $x^L \le y^L$ and $x^U \le y^U$.

Definition 2.13. We say that $[x] < [y]$ if $\begin{cases} x^L \le y^L \\ x^U < y^U \end{cases}$ or $\begin{cases} x^L < y^L \\ x^U \le y^U \end{cases}$ or $\begin{cases} x^L \le y^L \\ x^U \le y^U \end{cases}$.

Definition 2.13. (Ishibuchi and Tanaka,1990)

We say that $[x] \preccurlyeq [y]$ if $m([x]) \le m([y])$. If $x^U \le y^L$, the interval inequality relation "$[x] \preccurlyeq [y]$" is said to be optimistic satisfactory. If $x^U > y^L$, the interval inequality relation "$[x] \preccurlyeq [y]$"is said to be pessimistic satisfactory.

Definition 2.13. We say that $[x] \prec [y]$ if $x^L \le y^U$.

Definition 2.13. We say that $[x] \supset [y]$ if $x^L \le y^L$ and $x^U \ge y^U$.

3. INTERVAL LINEAR PROGRAMMING

A linear programming model with coefficients defined by real intervals is known as interval linear programming (ILP). Mathematically, following the approach from Allahdadi and Nehi (2011) and Chinneck and Ramadan, 2000, an ILP model can be stated as follows.

3.1. THE MATHEMATICAL MODEL

From the mathematical point of view, the model of a linear programming problem (ILP) has the following form:

$$(\text{ILP})\begin{cases} \max\ z = \sum_{j=1}^{n}[c_j]x_j \\ \text{such that}\quad \sum_{j=1}^{n}[a_{ij}]x_j \le [b_i],\ i=\overline{1,m_1} \\ \sum_{j=1}^{n}[d_{ij}]x_j \ge [b_{m_1+i}],\ i=\overline{1,m_2} \\ x_j \ge 0,\ j=\overline{1,n} \end{cases}$$

where x_j represent decision variables and expressions of type $[\alpha]$ are interval numbers $[\alpha]=[\alpha^L,\alpha^U]$.

An equivalent model as ILP is given by:

$$(\text{ILP})\begin{cases} \max\ z = \sum_{j=1}^{n}[c_j]x_j \\ \text{such that}\quad \left[\sum_{j=1}^{n}a_{ij}^L x_j,\ \sum_{j=1}^{n}a_{ij}^U x_j\right] \le \left[b_i^L,\ b_i^U\right],\ i=\overline{1,m_1} \\ \left[\sum_{j=1}^{n}d_{ij}^L x_j,\ \sum_{j=1}^{n}d_{ij}^U x_j\right] \ge \left[b_{m_1+i}^L,\ b_{m_1+i}^U\right],\ i=\overline{1,m_2} \\ x_j \ge 0,\ j=\overline{1,n} \end{cases}$$

3.2. ALGORITHM FOR SOLVING THE INTERVAL OPTIMIZATION PROBLEM

We propose an algorithm for solving the interval optimization problem, described as follows:

Stage 1. Determine the optimal "best" solution ($x^{(1)}$) by solving the following problem (considering the safest possible restrictions):

$$\begin{cases} \min z_1^{'} = \sum_{j=1}^{n} c_j^L x_j \\ \text{such that} \quad \sum_{j=1}^{n} a_{ij}^L x_j \le b_i^U, \ i = \overline{1, m_1} \\ \sum_{j=1}^{n} d_{ij}^U x_j \ge b_{m_1+i}^L, \ i = \overline{1, m_2} \\ x_j \ge 0, \ j = \overline{1, n} \end{cases}$$

Remark. If the goal is to maximize the objective function, the following problem must be solved:

$$\begin{cases} \max z_1^{''} = \sum_{j=1}^{n} c_j^U x_j \\ \text{such that} \quad \sum_{j=1}^{n} a_{ij}^L x_j \le b_i^U, \ i = \overline{1, m_1} \\ \sum_{j=1}^{n} d_{ij}^U x_j \ge b_{m_1+i}^L, \ i = \overline{1, m_2} \\ x_j \ge 0, \ j = \overline{1, n} \end{cases}$$

Stage 2. Determine the optimal "worst" solution ($x^{(2)}$) by solving the following problem (considering the strictest possible restrictions)

$$\begin{cases} \min z_2^{'} = \sum_{j=1}^{n} c_j^U x_j \\ \text{such that} \quad \sum_{j=1}^{n} a_{ij}^L x_j \le b_i^U, \ i = \overline{1, m_1} \\ \sum_{j=1}^{n} d_{ij}^U x_j \ge b_{m_1+i}^L, \ i = \overline{1, m_2} \\ x_j \ge 0, \ j = \overline{1, n} \end{cases}$$

Remark. If the goal is to maximize the objective function, the following problem must be solved:

$$\begin{cases} \max\ z_2^{''} = \sum_{j=1}^{n} c_j^L\ x_j \\ \text{such that} \quad \sum_{j=1}^{n} a_{ij}^U\ x_j \le b_i^L,\ i = \overline{1, m_1} \\ \sum_{j=1}^{n} d_{ij}^L\ x_j \ge b_{m_1+i}^U,\ i = \overline{1, m_2} \\ x_j \ge 0,\ j = \overline{1, n} \end{cases}$$

Solving the two linear programming problems presented above by classical methods (simplex algorithm), we can draw the following conclusions:

- For a minimization problem, the optimal range for the objective function is given by the interval number $[z_1^{'}, z_2^{'}]$.
- For a maximization problem, the optimal range for the objective function is given by the interval number $[z_1^{''}, z_2^{''}]$.
- The optimal solution of the optimization problem is given by the interval number $x^0 = \lambda x^{(1)} + (1-\lambda)x^{(2)}$, $\lambda \in [0,1]$.

4. PORTFOLIO OPTIMIZATION MODEL USING INTERVAL ANALYSIS

4.1. THE MTHEMATICAL MODEL

The optimization model based on interval analysis proposed in Jong, 2012, which uses semi-absolute deviation for risk assessment, presents some shortcomings from computational point of view. For this reason, in this paper we will use the absolute deviation of return, modeled using interval numbers.

In this section, in order to perform portfolio optimization, we will use a pattern of deviation where the future income assets are treated as interval numbers.

Suppose an investor wants to allocate a capital into n risky assets, with random returns. We introduce the following notations. We will denote by:

- T the time horizon of the investment;
- $\tilde{r}_j$ the interval average return of asset $j,\ j=\overline{1,n}$
- x_j the proportion of the total amount invested in asset $j,\ j=\overline{1,n}$;
- x_j^0 the proportion of the asset $j,\ j=\overline{1,n}$, already owned by the investor;
- r_{jt} the historical rate of return of asset j at time t, with $j=\overline{1,n},\ t=\overline{1,T}$;
- u_j the edge higher percentage of the capital invested in the asset $j,\ j=\overline{1,n}$.

The average rate of return of the asset $j,\ j=\overline{1,n}$, can be represented using the following interval numbers: $\tilde{r}_j=\left[r_j^L,\ r_j^U\right]=\left[\min\{r_{ja},r_{jh}\},\max\{r_{ja},r_{jh}\}\right]$, where:

- r_{ja} represents the arithmetic mean of the returns of asset $j,\ j=\overline{1,n}$ based on historical data, expressed as: $r_{ja}=\frac{1}{T}\sum_{t=1}^{T}r_{jt}$;
- r_{jh} represents the historical mean return tendency of the returns of asset $j,\ j=\overline{1,n}$; (if the rate of return of asset j have increased, it is acceptable that the rate of return is higher arithmetic performance based on historical data).

Then the average rate of return of the portfolio x is given by:

$$\tilde{r}(x)=\sum_{j=1}^{n}\tilde{r}_j\,x_j=\left[\sum_{j=1}^{n}r_j^L x_j,\ \sum_{j=1}^{n}r_j^U x_j\right].$$

Since the average returns of assets are modeled using interval numbers, we can quantify the deviation of the return of the portfolio x using an interval number:

$$\tilde{w}_t(x)=\left[w_t(x)^L,\ w_t(x)^U\right]=\left[\sum_{j=1}^{n}\min\left(\left|r_j^L-r_{jt}\right|,\left|r_j^U-r_{jt}\right|\right)x_j,\ \sum_{j=1}^{n}\max\left(\left|r_j^L-r_{jt}\right|,\left|r_j^U-r_{jt}\right|\right)x_j\right].$$

Then the mean deviation interval for the portfolio x is given by:

$$\tilde{w}(x)=\left[\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{n}\min\left(\left|r_j^L-r_{jt}\right|,\left|r_j^U-r_{jt}\right|\right)x_j,\ \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{n}\max\left(\left|r_j^L-r_{jt}\right|,\left|r_j^U-r_{jt}\right|\right)x_j\right]$$

We will use $\tilde{w}(x)$ to assess the risk of the portfolio risk x. Assume that an investor seeks to maximize the portfolio return at a given level of risk.

We will denote by w^L and w^U the lower, respective the higher level risk tolerated. If the tolerance interval $\tilde{w} = \left[w^L, w^U\right]$ for the risk of the portfolio is given, then the model of the portfolio optimization problem can be represented as follows:

$$\begin{cases} \max\ \tilde{r}(x) = \sum_{j=1}^{n} \tilde{r}_j x_j \\ \text{such that} \quad \tilde{w}(x) \le \left[w^L, w^U\right] \\ \sum_{j=1}^{n} x_j = 1 \\ 0 \le x_j \le u_j,\ j = \overline{1,n} \end{cases}$$

4.2. SOLVING THE OPTIMIZATION PROBLEM

The method which can be used for solving the optimization problem is described in detail in subsection 3.1. The above problems are reduced to the following two linear programming problems, which can be solved using the simplex algorithm. We propose the following algorithm.

Stage 1. Determine the optimal "best" solution $x^{(1)}$ by solving the following problem and considering the safest possible restrictions:

$$(PPL_{(1)}) \begin{cases} \max\ \tilde{r}(x) = \sum_{j=1}^{n} \tilde{r}_j x_j \\ \text{such that} \quad \tilde{w}(x) \le \left[w^L, w^U\right] \\ \sum_{j=1}^{n} x_j = 1 \\ 0 \le x_j \le u_j,\ j = \overline{1,n} \end{cases}$$

or

$$(PPL_{(1)})\begin{cases} \max\ r(x)^U = \sum_{j=1}^{n} r_j^U\, x_j \\ \text{such that}\quad \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{n}\min\left(\left|r_j^L - r_{jt}\right|,\left|r_j^U - r_{jt}\right|\right)x_j \le w^U \\ \sum_{j=1}^{n} x_j = 1 \\ 0 \le x_j \le u_j,\ j=\overline{1,n} \end{cases}$$

Stage 2. Determine the optimal "worst" solution $x^{(2)}$ by solving the following problem, obtained by considering the strictest possible constraints:

$$(PPL_{(2)})\begin{cases} \max\ r(x)^L = \sum_{j=1}^{n} r_j^L\, x_j \\ \text{such that}\quad w(x)^U \le w^L \\ \sum_{j=1}^{n} x_j = 1 \\ 0 \le x_j \le u_j,\ j=\overline{1,n} \end{cases}$$

or

$$(PPL_{(2)})\begin{cases} \max\ r(x)^L = \sum_{j=1}^{n} r_j^L\, x_j \\ \text{such that}\quad \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{n}\max\left(\left|r_j^L - r_{jt}\right|,\left|r_j^U - r_{jt}\right|\right)x_j \le w^L \\ \sum_{j=1}^{n} x_j = 1 \\ 0 \le x_j \le u_j,\ j=\overline{1,n} \end{cases}$$

We obtain the optimal solution given by: $x^0 = \lambda x^{(1)} + (1-\lambda)x^{(2)}$, $\lambda \in [0,1]$.

5. CASE STUDY

We consider the case of two assets listed at Bucharest Stock Exchange: asset 1 = TGN (Transgaz), asset 2 = FP (Fondul Proprietatea). We have used the closing prices of these assets corresponding to the time horizon 01.04.2014-25.09.2014.

$$r_{1a} = \frac{1}{T}\sum_{t=1}^{T} r_{1t}$$

$$r_{1h} = \frac{1}{\tau}\sum_{t=T-\tau+1}^{T} r_{1t}$$

$$r_{2a} = \frac{1}{T}\sum_{t=1}^{T} r_{2t}$$

$$r_{2h} = \frac{1}{\tau}\sum_{t=T-\tau+1}^{T} r_{2t}$$

$$\tilde{r}_1 = \left[\min\left(r_{1a}, r_{1h}\right), \max\left(r_{1a}, r_{1h}\right)\right] = \left[r_1^L,\, r_1^U\right]$$

$$\tilde{r}_2 = \left[\min\left(r_{2a}, r_{2h}\right), \max\left(r_{2a}, r_{2h}\right)\right] = \left[r_2^L,\, r_2^U\right]$$

We have used T = 124 days and τ = 10 days. We have obtained:

$r_{1a} = 0.0632,\ r_{1h} = 0.1847,\ r_{2a} = 0.0068,\ r_{2h} = 0.1110.$

It results:

$\tilde{r}_1 = \left[0.0632;\ 0.1847\right]$ and $\tilde{r}_2 = \left[0.0068;\ 0.1110\right]$.

We suppose that the investor acts rationally, so we chose $\tilde{w} = [0.02;\ 0.06]$ the tolerance interval and $u_1 = u_2 = 0.8$.

The optimization problem which must be solved is given by:

$$\begin{cases} \max\ \tilde{r}(x) = \tilde{r}_1\, x_1 + \tilde{r}_2\, x_2 \\ \text{such that}\quad \dfrac{1}{124}\displaystyle\sum_{t=1}^{124}\left(\left|\tilde{r}_1 - r_{1t}\right| x_1 + \left|\tilde{r}_2 - r_{2t}\right| x_2\right) \le [0.02;\ 0.06] \\ x_1 + x_2 = 1 \\ 0 \le x_j \le 0.8,\ j = \overline{1,2} \end{cases}$$

Solving this problem reduces to solving the optimization problems described in subsection 3.2. So we have to solve the following problems:

$$(\mathrm{PPL}_{(1)})\begin{cases} \max\ r(x)^U = r_1^U\ x_1 + r_2^U\ x_2 \\ \text{such that}\quad \dfrac{1}{124}\sum_{t=1}^{124}\Big[\min\Big(\big|r_1^L - r_{1t}\big|,\big|r_1^U - r_{1t}\big|\Big)x_1 + \min\Big(\big|r_2^L - r_{2t}\big|,\big|r_2^U - r_{2t}\big|\Big)x_2\Big] \le 0.6 \\ x_1 + x_2 = 1 \\ 0 \le x_j \le 0.8,\ j = \overline{1,2} \end{cases}$$

and

$$(\mathrm{PPL}_{(2)})\begin{cases} \max\ r(x)^U = r_1^U\ x_1 + r_2^U\ x_2 \\ \text{such that}\quad \dfrac{1}{124}\sum_{t=1}^{124}\Big[\max\Big(\big|r_1^L - r_{1t}\big|,\big|r_1^U - r_{1t}\big|\Big)x_1 + \max\Big(\big|r_2^L - r_{2t}\big|,\big|r_2^U - r_{2t}\big|\Big)x_2\Big] \le 0.2 \\ x_1 + x_2 = 1 \\ 0 \le x_j \le 0.8,\ j = \overline{1,2} \end{cases}$$

The "best" solution is given by the optimal solution of ($\mathrm{PPL}_{(1)}$), whereas the "worst" solution is given by the optimal solution of ($\mathrm{PPL}_{(2)}$). We obtain:
$x^{(1)} = (0.28; 0.72)$; $x^{(2)} = (0.22; 0.78)$.
Then the optimal solution is given by: $x^0 = \lambda x^{(1)} + (1-\lambda)x^{(2)}$, $\lambda \in [0,1]$.

6. CONCLUSIONS

In this paper a new approach to portfolio optimization problem is proposed. The model developed takes into account both uncertainty and the decision maker idea and for that is more practical in the real world than others. We use interval analysis approach to handle the imprecise data in financial markets. The case study presented illustrates the application of the proposed model and demonstrates the effectiveness of the designed algorithm for solving our model.

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